# Imputation and forecasting for Multi-Output Gaussian Process in Smart Grid

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## Problem

Data forecasting is a key component of intelligent upgrading of power systems. However, Data obtained from the real world may have varying degrees of missing data. These missing components have a significant impact on the outcome of the prediction model [1]. In addition, data from different devices in the same region can significantly correlate for multiple prediction tasks. We study how to establish a single model for multiple datasets forecasting with missing data, which can not improve the accuracy of data forecasting.

## Basic Concepts

In a multi-output learning problem of $M$ tasks, a MOGP model can be taken to deal with $M$ tasks simultaneously [2]. Given $M$ tasks, the covariance matrix $K$ for $\mathbf{f}$ can be expressed as

$$K = K_m(\mathbf{M}, \mathbf{w}_m) \otimes K_t(\mathbf{X}, \mathbf{w}_t)$$

$K_m(\cdot)$ and $K_t(\cdot)$ are the covariance matrix between tasks and the covariance matrix between the corresponding inputs, respectively. $\mathbf{M} = \{m|m = 1, \cdots, M\}$ and $\mathbf{X} = \{\mathbf{x}_m|m = 1, \cdots, M\}$.

## Kernel Reconstruction

In this paper we utilize the linear model of coregionalization (LMC) model to achieve data imputation and prediction simultaneously [3]. The covariance matrix $K$ of $\mathbf{f}$ corresponding to the LMC model takes the form as follows:

$$K = \sum_{q=1}^{Q} K_m^q(\mathbf{M}, \mathbf{w}_m) \otimes K_t^q(\mathbf{X}, \mathbf{w}_t)$$

In addition, we consider to design a mixture of two frequently used covariance functions, Rational Quadratic (RQ) and Periodic (PE) covariance functions, to capture the periodicity and smoothness of energy data to vary over longer distances.

$$K_{RP}(x, x') = K_{RQ} * K_{PE}$$

$$= \sigma^2 \Big(1 + \frac{(x - x')^2}{2\alpha \ell_{RQ}^2}\Big)^{-\alpha} \exp\Big(-\frac{2sin^2(\pi|x - x'|/p)}{\ell_{PE}^2}\Big)$$

## References

[1] Ioannis P. Panapakidis, Aggelos S. Bouhouras, and Georgios C. Christoforidis, *A missing data treatment method for photovoltaic installations*, In 2018 IEEE International Energy Conference(ENERGYCON), pages 1–6, 2018.

[2] Carl Edward Rasmussen and Christopher K. I. Williams, *Gaussian processes for machine learning.* 2006.

[3] Hao Zhang, *Maximum-likelihood estimation for multivariate spatial linear coregionalization models*, Environmetrics, 18(2):125–139, (2007)

[4] EDF energy company, *EDF's open data platform on the Ponant Islands*, https://opendata-iles-ponant.edf.fr/pages/home

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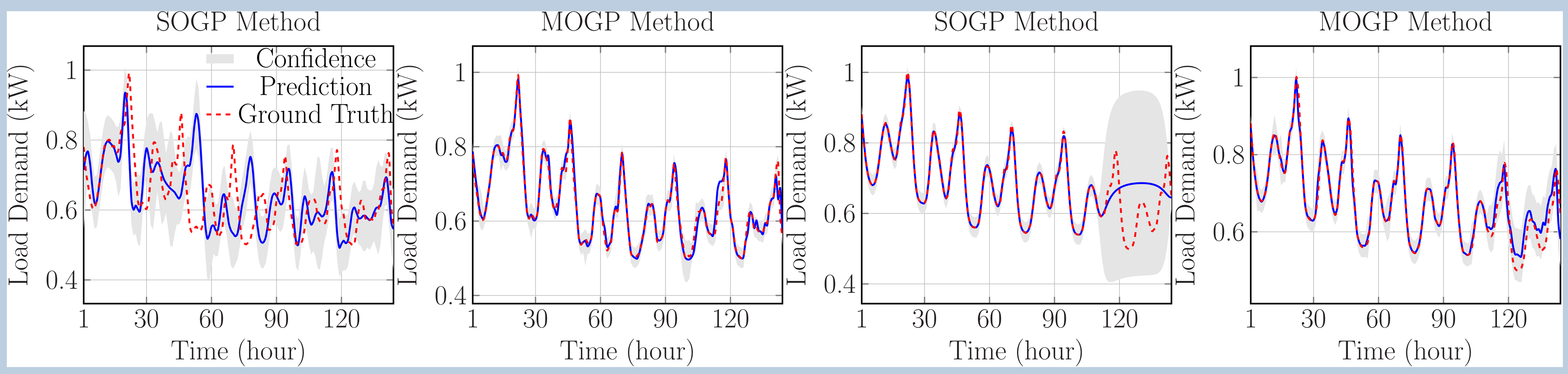
## Results and Discussion

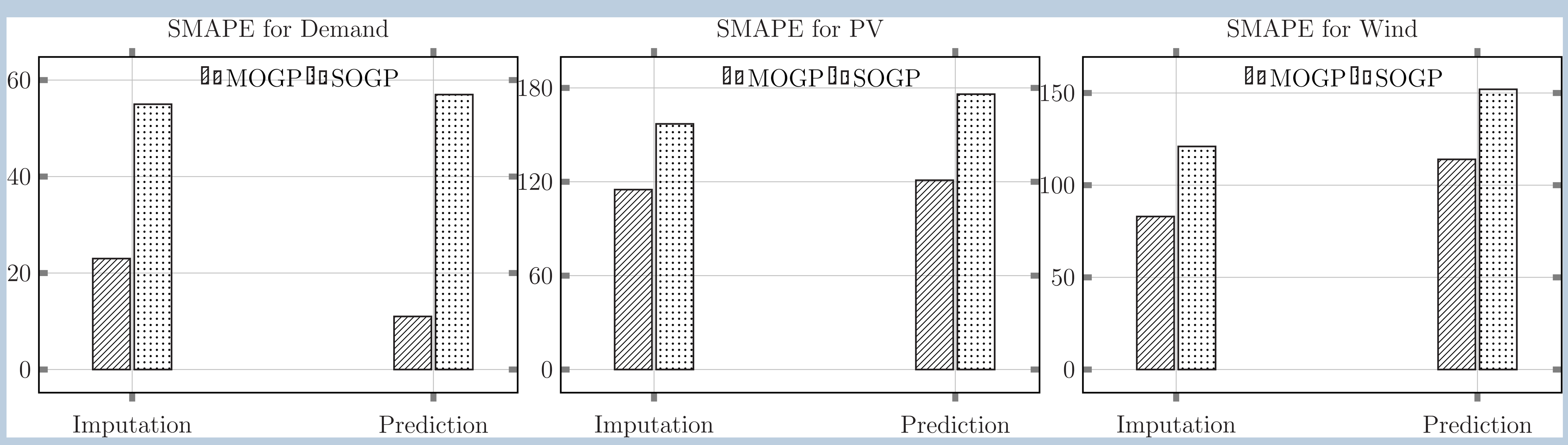
Our experimental simulation considers the energy datasets for isolated power system from three islands including Ushant, Lundy and Isles of Scilly based on the EDF energy company [4]. The historical data of wind generation, PV generation, and load demand for a period of time, sixty days are used to constitute the training datasets while the forecasting performance of different tasks is validated for a period of 24 hours.

Table 1: Comparison results of MSE values obtained by SOGP and MOGP methods for imputation and forecasting

| Approach | Demand | PV | Wind |
|---|---|---|---|
| Imputation (1440*3) | | | |
| SOGP | 7.147E-1(1.58E-2) | 6.975E-1(1.42E-4) | 6.557E-1(9.06E-3) |
| MOGP | 4.114E-2(7.51E-3) | 5.058E-2(6.78E-5) | 1.271E-1(7.65E-3) |
| Prediction (1440*3) | | | |
| SOGP | 7.469E-1(9.58E-3) | 9.157E-1(1.27E-3) | 8.491E-1(9.64E-2) |
| MOGP | 2.132E-3(7.98E-9) | 1.324E-1(1.87E-7) | 8.759E-2(4.98E-8) |




As you can see from the figure above, we can see that using the MOGP method has a significant impact on the performance of all three types of energy data compared with SOGP method(Left two for imputation, right two for forecasting), especially the predicted results.




To have a better investigation for the performance difference achieved by using the standard SOGP method against the MOGP method, we also show the comparison SMAPE results for wind, PV, and load demand. From this result, it is clear to see that the performance has been greatly improved, especially the prediction results for demand.

Table 2: Comparison results obtained by different methods

| Type | MTLGPTS | MTBSGP | MOGP |
|---|---|---|---|
| Imputation MSE Values | | | |
| Demand | 1.062E-1(1.38E-3) | 1.833E-1(9.20E-5) | **3.984E-2(2.61E-4)** |
| PV | 4.316E-1(2.57E-2) | 3.435E-1(2.20E-3) | **5.817E-2(3.35E-4)** |
| Wind | 2.715E-1(7.43E-4) | 2.297E-1(1.41E-3) | **1.436E-1(6.33E-4)** |
| Imputation SMAPE Values | | | |
| Demand | 4.587E+1(7.24E-0) | 4.746E+1(1.17E+1) | **3.621E+1(3.41E-0)** |
| PV | 4.985E+1(14.58E+1) | 5.243E+1(8.25E-0) | **4.038E+1(2.95E-0)** |
| Wind | 5.357E+1(2.25E+1) | 6.125E+1(8.65E-0) | **4.259E+1(5.52E-0)** |
| Prediction MSE Values | | | |
| Demand | 9.772E-2(9.56E-3) | 9.569E-2(4.24E-4) | **2.035E-3(3.42E-5)** |
| PV | 6.263E-1(5.47E-3) | 5.346E-1(5.84E-4) | **1.372E-1(6.54E-4)** |
| Wind | 3.552E-1(9.23E-3) | 3.254E-1(6.21E-4) | **8.534E-2(4.02E-5)** |
| Prediction SMAPE Values | | | |
| Demand | 5.447E+1(8.68E-0) | 5.236E+1(1.85E+1) | **3.423E+1(1.54E-0)** |
| PV | 6.056E+1(1.96E+1) | 5.971E+1(1.78E+1) | **4.098E+1(1.42E+1)** |
| Wind | 5.127E+1(2.47E+1) | 5.972E+1(1.16E+1) | **4.018E+1(3.57E-0)** |

In contrast, by being equipped with our proposed MOGP method, the imputation and forecasting performance have been improved against the SOGP method and other two state-of-the-art methods in Table 2.