# Variational Bayesian Inference and Learning for Continuous Switching Linear Dynamical Systems

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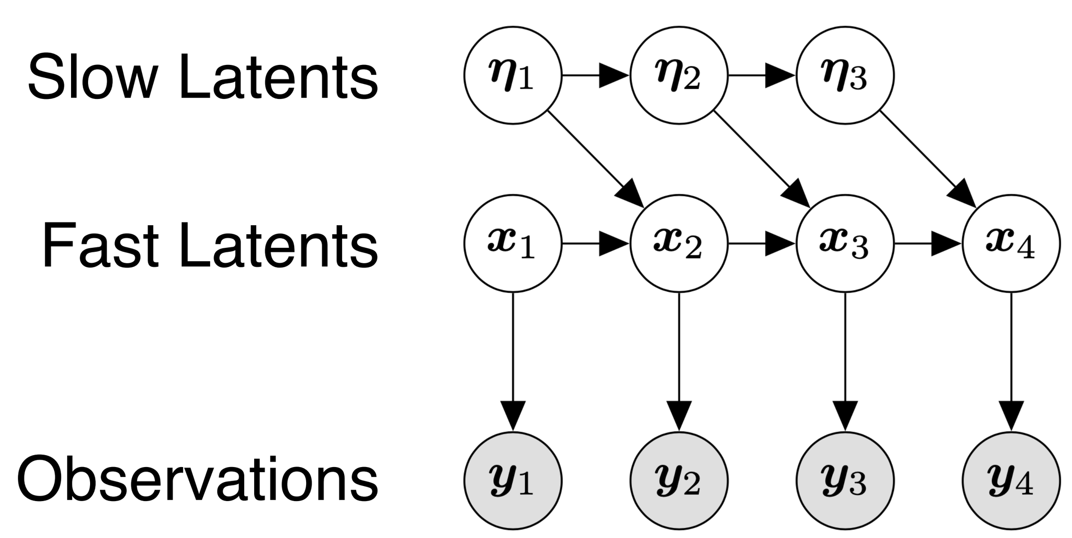
## Why a continuous SLDS?

Switching linear dynamical systems (SLDSs) posit a discrete number of distinct linear regimes with Markov transitions between regimes. We propose the continuous switching linear dyanmical system (CSLDS) for modeling data that are better described by dynamical regimes that are modulated continuously in time, for example by the dynamical regimes exhibited by neural circuits modulated by continuously-varying neuromodulator concentrations.

## Model description




$$
\begin{aligned}
\boldsymbol{\eta}^{(k)}_{1:T-1} &\sim \mathcal{GP}(\mathbf{0}, \kappa_k) \\
\boldsymbol{x}_1 &\sim \mathcal{N}(0, Q_{\mathrm{IC}}^{-1}) \\
\boldsymbol{x}_t | \boldsymbol{\eta}_{t-1}, \boldsymbol{x}_{t-1} &\sim \mathcal{N}(A(\boldsymbol{\eta}_{t-1})\boldsymbol{x}_{t-1}, [Q(\boldsymbol{\eta}_{t-1})]^{-1}) \\
\boldsymbol{y}_t | \boldsymbol{x}_t &\sim \mathcal{N}(C\boldsymbol{x}_t + \boldsymbol{b}, R)
\end{aligned}
$$

*where*

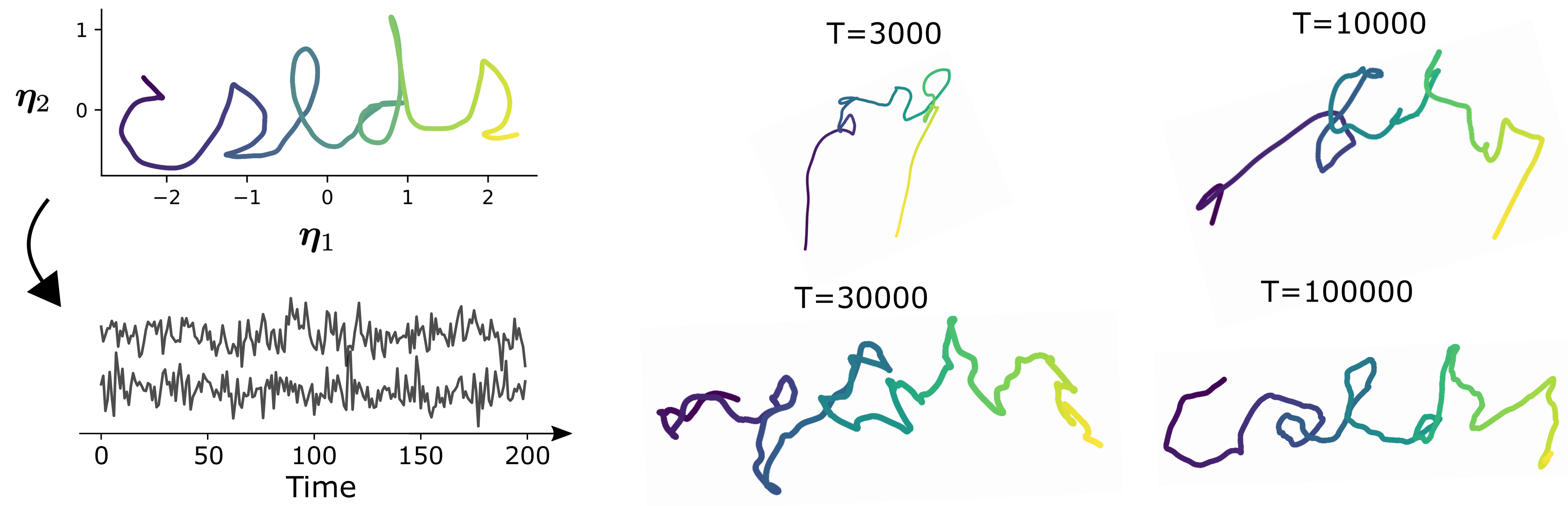
*"factor" definition*
$$A(\boldsymbol{\eta}) = A^{(0)} + \textstyle\sum_{k=1}^{K} \eta^{(k)} A^{(k)}\ , \quad \boldsymbol{s} = \mathrm{Softplus}(B\boldsymbol{\eta} + \boldsymbol{c})\ , \quad [Q(\boldsymbol{\eta})]^{-1} = \textstyle\sum_{k=1}^{K} s^{(k)} (Q^{(k)})^{-1}$$

*"simplex" definition*
$$\boldsymbol{s} = \mathrm{Softmax}(B\boldsymbol{\eta} + \boldsymbol{c})\ , \quad A(\boldsymbol{\eta}) = \textstyle\sum_{k=1}^{K} s^{(k)} A^{(k)}\ , \quad [Q(\boldsymbol{\eta})]^{-1} = \textstyle\sum_{k=1}^{K} s^{(k)} (Q^{(k)})^{-1}$$

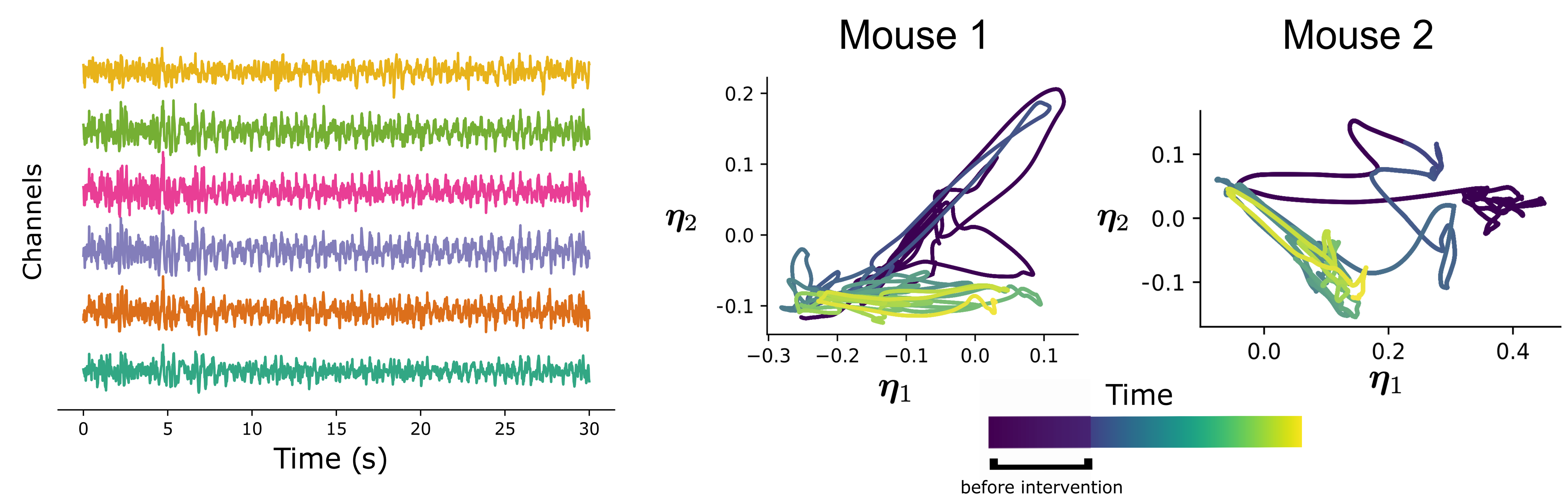
**ELBO objective:**

$$\mathcal{L}(\boldsymbol{y}_{1:T}; \theta, \phi) \triangleq \mathbb{E}_{\boldsymbol{\eta}_{1:T-1} \sim q(\boldsymbol{\eta}_{1:T-1})} \log \left[ \frac{p(\boldsymbol{y}_{1:T}, \boldsymbol{\eta}_{1:T-1})}{q(\boldsymbol{\eta}_{1:T-1})} \right] \leq \log p(\boldsymbol{y}_{1:T})$$

## Identifiability of the slow latents




## Application to mutlichannel neural data




## Some future directions

1) Fitting the model via approximate EM with Jax
2) Benchmark CSLDS versus SLDS models
3) Looking for a variety of neural & behavioral datasets