Uncovering the short-term dynamics of electricity day-ahead markets

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Electricity prices follow a multivariate generalised Langevin equation with drift and diffusion terms learnt from historical data

Stochastic representation of electricity prices

Market structure: Electricity day-ahead markets are typically split into 24 intraday time ticks (ITT), one ITT per day hour. The cleared price for each ITT is the electricity spot price.

State-of-the-art stochastic models: Current models analyse spot price as univariate time series[1]. Explicit terms are introduced ad hoc to account for spot price features, e.g., Ornstein-Uhlenbeck (OU) process to incorporate mean-reversion effects[2, 3].

Multivariate approach: The IITs structure needs a multivariate formulation to better represent the distinct price dynamics for each ITT.

A data-driven multivariate stochastic model

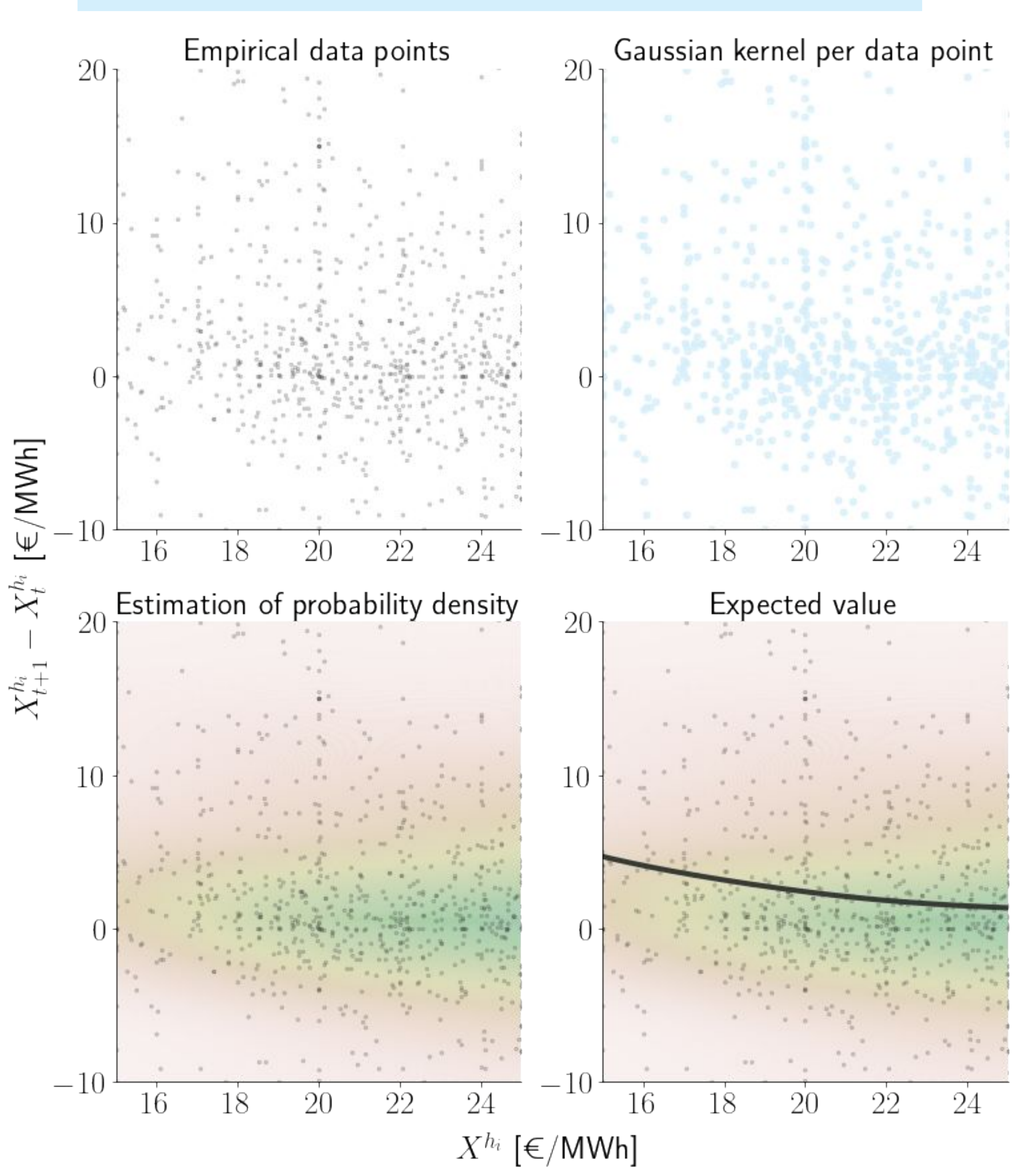
$$dX_t^{h_i} = \mu^{h_i}(\mathbf{X}_t)dt + \sigma_{h_i h_j}(\mathbf{X}_t)dW_t^{h_j}$$

Drift and diffusion terms: Obtained from the definitions of the Kramers-Moyal (KM) coefficients[4].

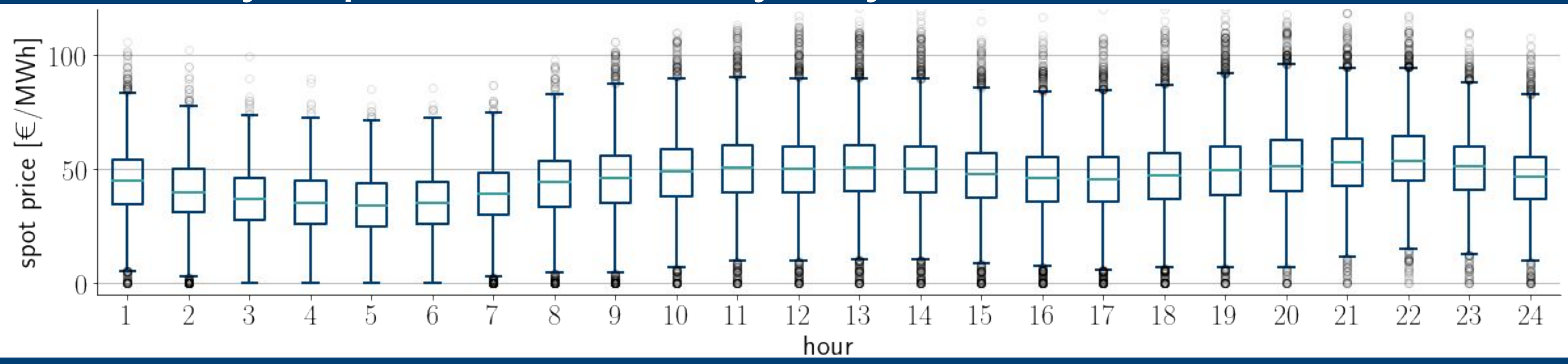
$$D_{h_i}^{(1)}(\mathbf{X}) = \lim_{\tau \to 0} \frac{\langle X_{t+\tau}^{h_i} - X^{h_i} \rangle}{\tau} = \mu^{h_i}(\mathbf{X})$$

$$D_{h_i h_j}^{(2)}(\mathbf{X}) = \frac{1}{2} \lim_{\tau \to 0} \frac{\langle [X_{t+\tau}^{h_i} - X^{h_i}][X_{t+\tau}^{h_j} - X^{h_j}] \rangle}{\tau} = \frac{1}{2}\sigma_{h_i h_k}(\mathbf{X})\sigma_{h_j h_k}(\mathbf{X})$$

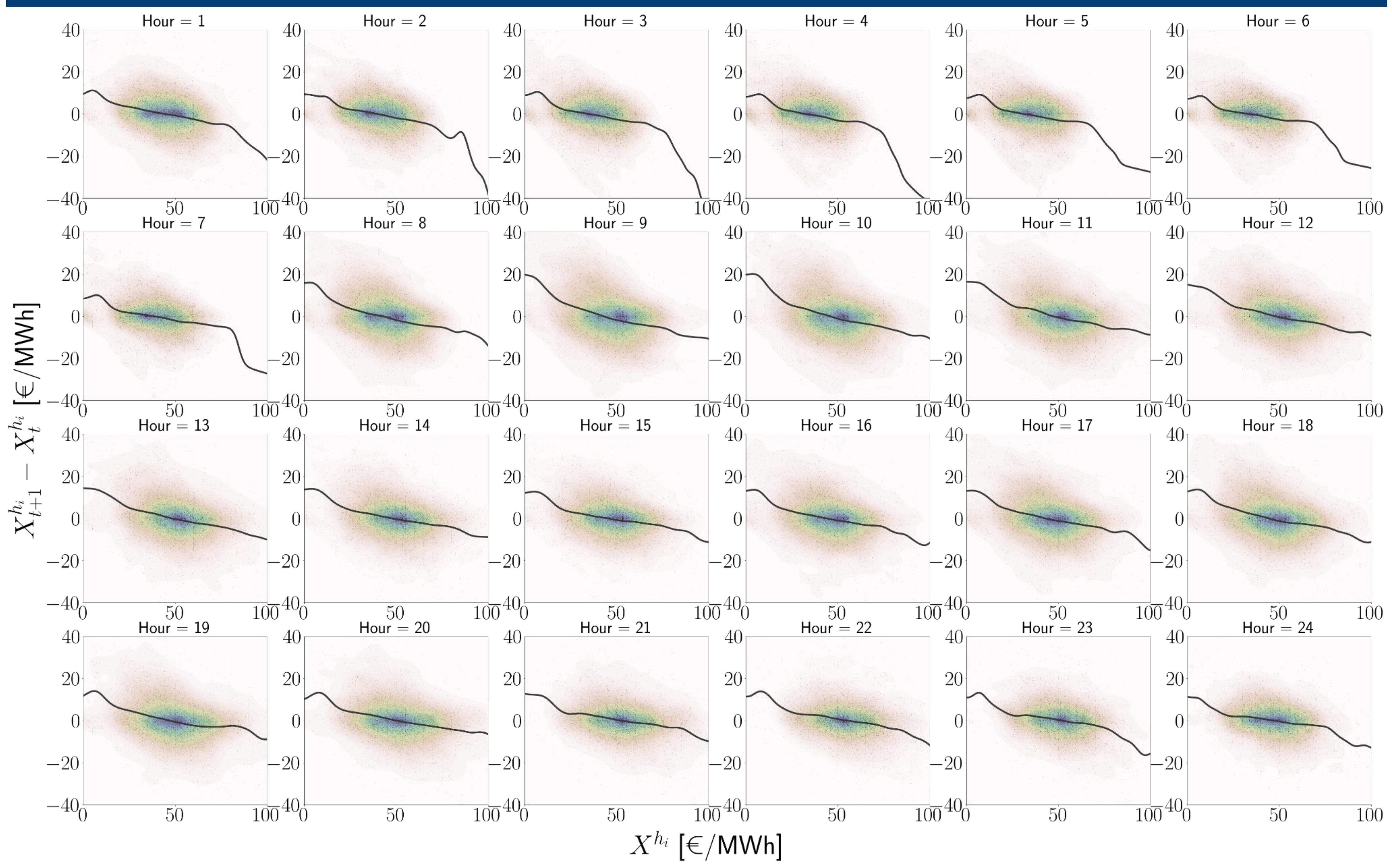
Expected values: Historical probability measure is computed by using a Gaussian kernel density estimation (KDE).



Case study: Spanish electricity day-ahead market 2004-2020



Drift function: first KM coefficient

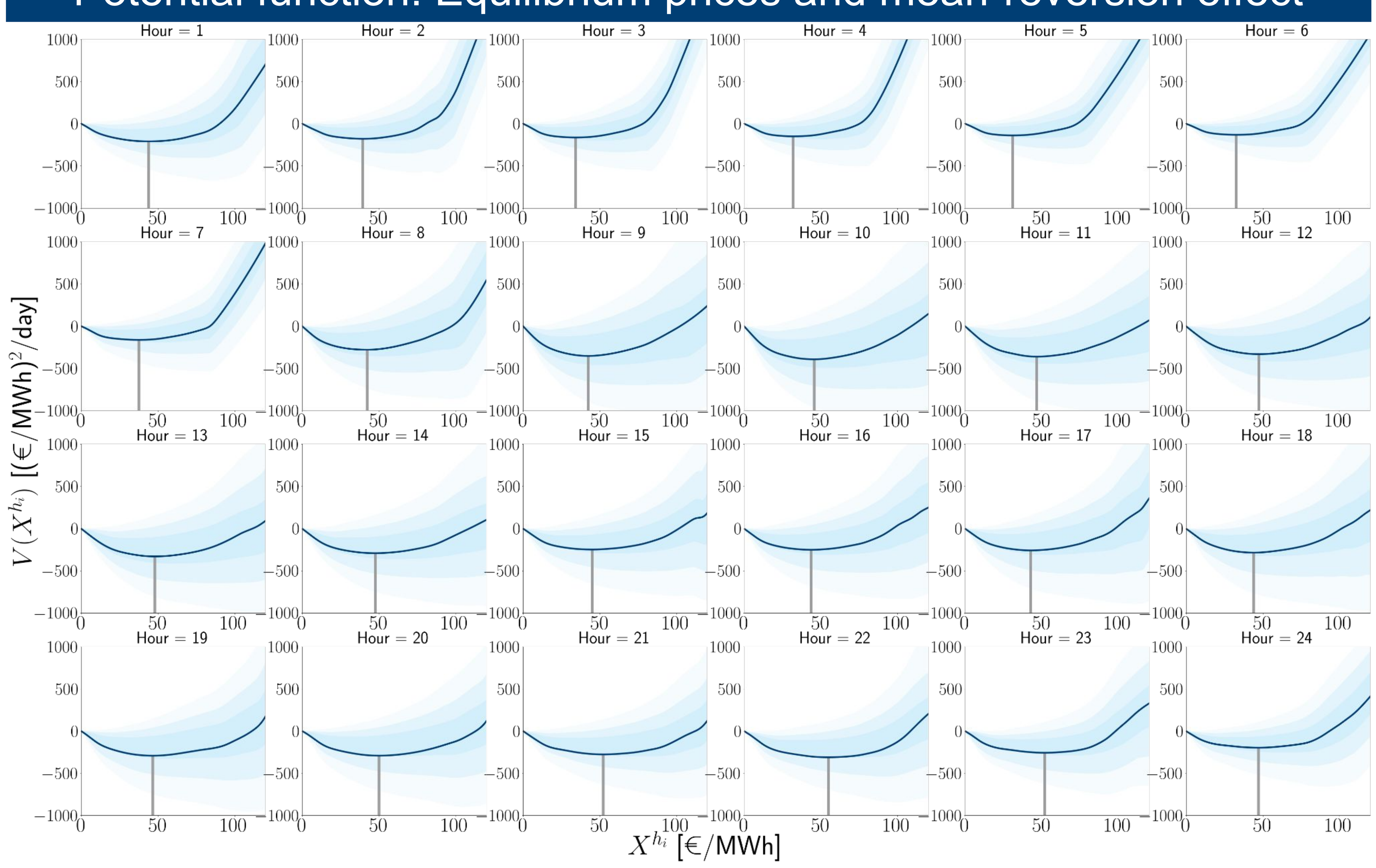


KDE as a function of the spot price and spot-price difference, for each ITT. Black solid lines represent the first KM coefficient extracted from the estimated PDF.

Extension of classical OU processes: Drift as a function of price and ITT instead of a single mean-reversion parameter.

Nonlinear price dynamics: Nonlinear dynamics arise from historical data without prior assumptions.

Potential function: Equilibrium prices and mean-reversion effect



Potential function for each ITT. Dark-blue solid lines represent the expected potential. The grey vertical lines indicate the minimum of the expected potential.

Equilibrium prices: Parabolic shape for each ITT, although the minimum varies amongst ITTs.

Mean-reversion effect: Reminiscent of a particle fluctuating around a harmonic potential.

References

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