Uncovering the short-time dynamics of electricity day-ahead markets

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Abstract

Obtaining a mathematical representation of electricity market prices is the cornerstone of the decision-making process in a liberalised landscape. Most of the existing models analyse the day-ahead electricity price as a univariate time series. This approach requires prior assumptions on the mathematical formulation to obtain an accurate representation of the time-evolution of electricity prices. We propose a new multivariate-stochastic-process model for the day-ahead prices, with each dimension of the process representing a single intraday time tick (ITT) auctioned in the day-ahead market. In this model, the electricity price at each ITT is the solution of a stochastic differential equation (so-called generalised Langevin equation) with the drift and diffusion terms to be learnt from historical data. The terms governing the stochastic differential equation are obtained by a kernel density estimation of the historical probability, to compute the expected values involved in the Kramers-Moyal definitions. The model is tested using data of the Spanish electricity day-ahead market, yielding a reliable representation of the electricity price structure. The price structure reveals the main features commonly discussed in electricity markets, such as the mean-reversion or equilibrium prices. Our results help us to understand the underlying short-time dynamics that govern the electricity day-ahead markets.

1 Extended abstract

The continuous liberalisation of electricity markets aims to enhance market participation and economic efficiency while fostering the integration of renewable energy in the system. Since the market design and market players have changed over the last few years, the deregulation has directly impacted on electricity prices, modifying both price levels and volatility. Not surprisingly, the time-evolution of electricity price has become a signal of market behaviour. Thus, the analysis and modelling of the electricity price has gathered considerable attention for market players as well as serving as a decision-making tool for policymakers. Examples of the modelling of electricity prices arise in fields as diverse as risk management [1], electricity price forecasting [6], or price simulation [4].

Among the existing electricity markets, the day-ahead market governs the bulk of the electricity system operation. Typically, the day-ahead market is held one day before delivery, when demand and generator agents submit their offers for each intraday time tick (ITT) of the following day. A common configuration of the day-ahead market consists of 24 ITTs (one per day hour). The offers represent the price and amount of energy that the agents are willing to consume, or produce, for a given ITT. The offers are ranked for both demand and generator agents and the intersection provides the cleared price for every ITT, usually referred as the spot price.

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Stochastic processes represent a natural mathematical setting to model fluctuating time series, as is the case of the electricity spot price. State-of-the-art stochastic models [5] describe the random behaviour of electricity spot prices relying on Brownian motions (Gaussian models) or extended formulations (Lévy processes, regime-switching frameworks or stochastic volatility models). With these formulations, the resulting time-evolution of the electricity price captures the most widely-know features of the spot price including seasonal patterns, mean-reversion effect and price spikes [8]. Often, this feature representation is the result of an imposed bias on the model where an explicit term is commonly introduced in the mathematical formulation to account for the desired feature. For instance, the Ornstein-Uhlenbeck (OU) process [14] and its extensions are widely applied to represent electricity prices [10, 7, 2], due to the OU process ability to incorporate mean-reversion effects. Besides stochastic processes, autoregressive models are also applied while studying spot prices. These autoregressive models use previous time records of the spot price time series as input features to forecast the time-evolution of the spot price. Within autoregressive models, we can find linear regressions [15], ARIMA family [3] and other machine learning related models [9]. The majority of stochastic and autoregressive models consider the electricity spot price as a univariate time series. This univariate approach might dilute important interactions between the prices at different IITs. This interaction effect is studied in Ref. [16], with a set of linear regressions for the spot price (one regression for each hourly spot price). Although, this work provides a multivariate formulation of the electricity spot price, the simplicity of linear regressions might mask nonlinear price dynamics that should be considered.

Regarding the existing methodologies, the need of a unifying bridge capable of considering the general multivariate and stochastic nature of the process becomes apparent. We propose a multivariate stochastic model where the drift and diffusion components are completely general and to be obtained from historical data by utilising the definitions of the Kramers-Moyal (KM) expansion coefficients [11]. Hence, the drift and diffusion coefficient are related to the expected value of the historical probability measure of the first and second KM coefficient, respectively. The historical probability measure is computed using a kernel density estimation (KDE) [13]. The mathematical formulation of our multivariate stochastic process is as follows:

$$dX_t^{h_i} = \mu^{h_i}(\mathbf{X}_t)dt + \sigma_{h_i h_j}(\mathbf{X}_t)dW_t^{h_j} \tag{1}$$

where $X_t^{h_i}$ is the spot price at hour h_i of day $t \in \mathbb{N}$ (with $h_i = \{1, \dots 24\}$), the drift μ^{h_i} and diffusion $\sigma_{h_i h_j}$ terms (with $h_j = \{1, \dots 24\}$) depend on the multivariate stochastic process level $\mathbf{X}_t = \{X_t^{h_i}\}$ and $W_t^{h_j}$ is a Wiener process vector. The relationship between the KM coefficients and the drift and diffusion is presented here, see the Appendix for a comprehensive derivation:

$$\begin{aligned} D_{h_i}^{(1)}(\mathbf{X}, t) &= \lim_{\tau \to 0} \frac{\langle X_{t+\tau}^{h_i} - X^{h_i} \rangle}{\tau} = \mu^{h_i}(\mathbf{X}) \\ D_{h_i h_j}^{(2)}(\mathbf{X}, t) &= \frac{1}{2} \lim_{\tau \to 0} \frac{\langle [X_{t+\tau}^{h_i} - X^{h_i}][X_{t+\tau}^{h_j} - X^{h_j}] \rangle}{\tau} = \frac{1}{2} \sigma_{h_i h_k}(\mathbf{X}) \sigma_{h_j h_k}(\mathbf{X}) \end{aligned} \tag{2}$$

where $D^{(1)}$ and $D^{(2)}$ are the first and second KM coefficients and $\langle \cdot \rangle$ is the shorthand notation for the conditional expectation: $\langle X_{t+\tau} - X_t \rangle = \mathbb{E}[X_{t+\tau} - X_t \,|\, X_t = y]$. Our formulation decouples the spot price time series into several processes, each one for every ITT of the market. While the state-of-the-art stochastic process commented above have constant drift and diffusion parameters, our model assumes that each hour has its own governing dynamics (μ^{h_i}) depending on the process level that might be affected by the rest of the hours, given that the diffusion $\sigma_{h_i h_j}$ yields an explicit interaction among hours.

The proposed multivariate stochastic model in Eq. (1) is applied to the Spanish electricity day-ahead market, considering the hourly electricity spot price time series that spans from January 1st, 2004 to December 31th 2020. See Apendix, Figure 4 for a boxplot of the hourly distribution of the spot price for the 24 periods of the Spanish day-ahead market, where the x-axis is the hour of the day and the y-axis is the spot price in €/MWh.

The first KM coefficient requires an estimation of the joint probability of the random variable $Y^{h_i} = (X^{h_i}, \Delta X^{h_i})$, where ΔX^{h_i} is the spot price difference between consecutive time steps at hour h_i, in this study the difference between consecutive days $\Delta X^{h_i} = X_{t+1}^{h_i} - X_t^{h_i}$. For the second KM coefficient, the joint probability of the random variable to approximate is

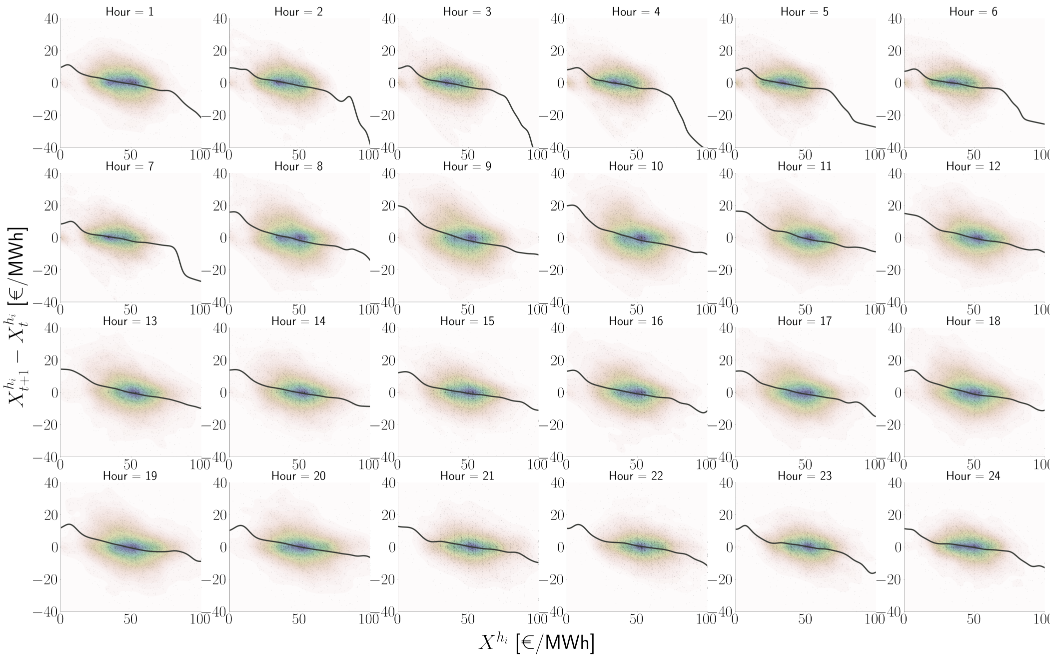

Figure 1: KDE of spot price - spot price difference for each hour. The coloured shadows represent the estimated probability, ranging from red colours (low probability) to blue colours (higher probability). The black solid line represents the first KM coefficient obtained from the estimated probability measure as a function of the spot price value.

$Z^{h_ih_j} = \left(X^{h_i}, X^{h_j}, \frac{1}{2}\Delta X^{h_i}\Delta X^{h_j}\right)$, with $\Delta X^{h_i}\Delta X^{h_j}$ representing the multiplication of spot price differences between consecutive time steps for hours h_i and h_j. A Gaussian kernel type is used in the KDE technique to approximate the historical joint probability measure of the aforementioned random variables. The bandwidth matrix is determined by the Scott's Rule [12].

Figure 1 shows the KDE results for Y^{h_i} together with the first KM coefficient. Recall that this first KM coefficient is equivalent to the drift term as stated in Eq. (2). We can observe how the drift term, i.e., the expected instantaneous price difference, changes depending on each hour and with respect to the spot price level even within the same hour. Although a similar pattern can be recognised in the drift of all hours, the intensities and local shapes of the drift are different amongst the hours.

To obtain a clearer understanding of the computed first KM coefficient (or drift), we assume the drift can be expressed as a conservative force, therefore as the gradient of a potential V: $D^{(1)}_{h_i}(\mathbf{X}) = \mu^{h_i}(\mathbf{X}) \doteq -\frac{dV(X^{h_i})}{dX^{h_i}}$. Figure 2 represents the computed potential function $V(X^{h_i})$ as a function of the spot price X^{h_i}. Noteworthy, the expected potential (dark blue line) exhibits a parabolic shape for every single hour. This means that the spot price has a certain equilibrium value (minimum of the parabola) towards which the current spot price is pushed with an either positive or negative force (depending on the slope of the potential which indicates the sign and return intensity). The return to the equilibrium value is alike the mean-reverting process that the OU represents in traditional stochastic processes. However, our approach allows us to define 24 different mean or equilibrium values, depending on the hour, instead of the single mean value that the univariate family model fits. With our formulation, the mean-reverting “velocity" parameter is a function of the price, whereas in classical OU processes there is one single mean-reverting velocity parameter regardless of the stochastic process value. While classical stochastic processes (like OU) for electricity price modelling impose the mean-reverting feature and adjust parameters to fit electricity prices, in our study, the mean-reverting feature and its dependency with the hour and spot price value have arisen from a pure data-driven technique applied to empirical electricity price observations.

Figure 3 depicts the second KM coefficient matrix for hours h_i and h_j in vertical and horizontal axis respectively. Each cell (h_i, h_j) contains the result of the $D^{(2)}_{h_ih_j}$ from Eq. (2) which is related to the

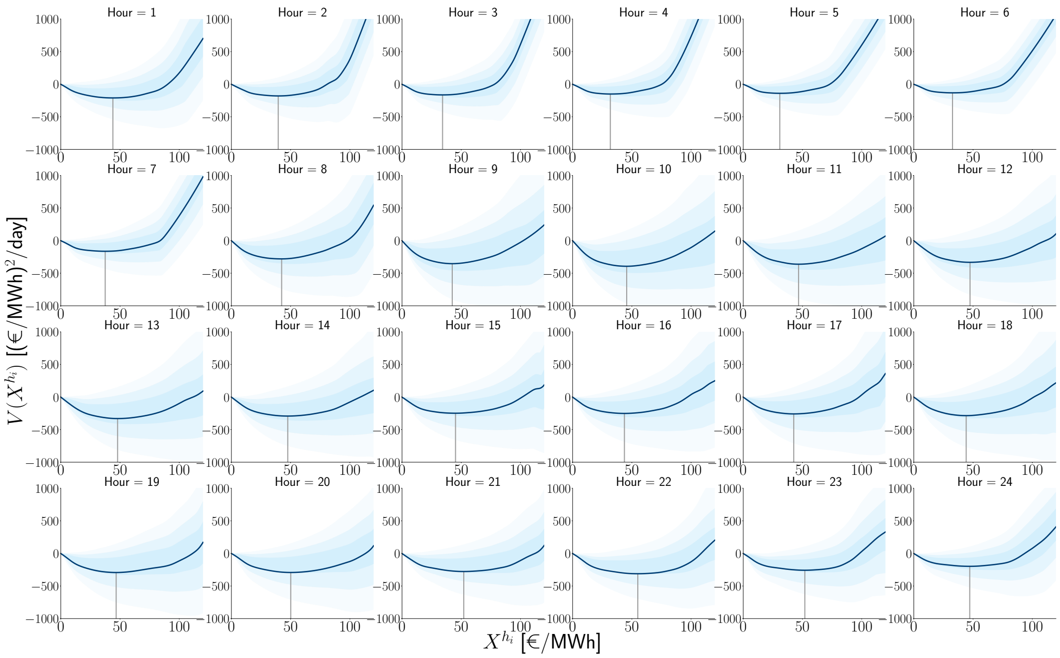

Figure 2: Potential function of each hour. The dark blue line represents the expected potential. The shaded areas delimit the potential percentiles. Starting from darkest to lightest areas: 40th-60th, 30th-70th and 20th-80th percentiles. Thus, the bottom lightest area enclose the 70th-80th percentiles, while the top lightest area enclose the 20th-30th percentiles. The vertical line indicates the minimum of the mean potential function (the equilibrium prices).

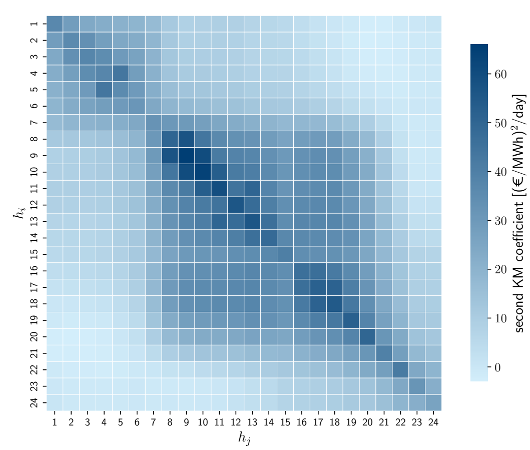

Figure 3: Second Kramers-Moyal coefficients obtained as the mean of the historical probability measure approximated by the KDE technique.

diffusion terms $\sigma_{h_i h_j}$. It is a symmetric matrix whose highest elements are positive and located in the main diagonal as given by definition. Besides, there are some hour clusters (higher second KM coefficient value) that reveals significant interactions between hours, as if they were correlated.

The particularity of our framework is that the components of the time evolution equations governing the dynamics of the prices are learnt directly from data, without the need of any prior model of the drift or the diffusive component, as is the case in traditional models. Future work would be devoted to solve the related stochastic differential equation in Eq. (1) with a numerical scheme. The objective would be to guarantee the model yields a reliable representation of the price structure and fluctuations of the Spanish electricity day-ahead market.

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A Appendix

Kramer-Moyal coefficients Considering a stochastic process $\mathbf{X}_t$ as in Eq. (1), $\mathbf{X}_t$ has an underlying probability density function (PDF), $\rho(\mathbf{X}, t)$, to observe a set of values $\mathbf{X} = \{X^{h_i}\}$ at time t. It can be shown that this PDF obeys the KM forward expansion [11]:

$$\frac{\partial \rho(\mathbf{X}, t)}{\partial t} = \sum_{n=1}^{\infty} \frac{(-\partial)^n}{\partial X^{h_{k_1}} \dots \partial X^{h_{k_n}}} D^{(n)}_{k_1,\dots,k_n}(\mathbf{X}, t)\rho(\mathbf{X}, t) \tag{3}$$

where $k_i \in \{1, \dots, 24\}$, $D^{(n)}_{k_1,\dots,k_n}(\mathbf{X}, t)$ is the n-th KM coefficient defined as follows:

$$D^{(n)}_{k_1,\dots,k_n}(\mathbf{X}, t) = \frac{1}{n!} \lim_{\tau \to 0} \frac{1}{\tau} \langle [X^{h_{k_1}}_{t+\tau} - X^{h_{k_1}}] \cdots [X^{h_{k_n}}_{t+\tau} - X^{h_{k_n}}] \rangle \tag{4}$$

where $\langle\cdot\rangle$ is the shorthand notation for the conditional expectation:

$$\langle X_{t+\tau} - X_t \rangle = \mathbb{E}^y[X_{t+\tau} - X_t \mid X_t], \tag{5}$$

with $\mathbb{E}^y[\cdot] = \mathbb{E}[\cdot \mid X_0 = y]$, for the fixed initial condition $y \in \mathbb{R}$. The KM coefficients defined in Eq. (4) govern the time-evolution equation given in (3), hence the stochastic process. To find the relationship between the KM coefficients and the drift and diffusion terms of Eq. (1) we consider the Stieltjes integral on a time interval of length $\tau > 0$, and the initial condition $X_t^{h_i} = X^{h_i}$:

$$X_{t+\tau}^{h_i} - X^{h_i} = \int_t^{t+\tau} \mu^{h_i}(\mathbf{X}_{t'})dt' + \int_t^{t+\tau} \sigma_{h_i h_j}(\mathbf{X}_{t'})dW_{t'}^{h_j} \tag{6}$$

We can now use the Taylor expansion of $\mu^{h_i}(\mathbf{X}_{t'})$ and $\sigma_{h_i h_j}(\mathbf{X}_{t'})$:

$$\begin{aligned}
\mu^{h_i}(\mathbf{X}_{t'}) &= \mu^{h_i}(\mathbf{X}) + \left(\frac{\partial}{\partial X^{h_k}}\mu^{h_i}(\mathbf{X})\right)(X_{t'}^{h_k} - X^{h_k}) + \ldots \\
\sigma_{h_i h_j}(\mathbf{X}_{t'}) &= \sigma_{h_i h_j}(\mathbf{X}) + \left(\frac{\partial}{\partial X^{h_k}}\sigma_{h_i h_j}(\mathbf{X})\right)(X_{t'}^{h_k} - X^{h_k}) + \ldots
\end{aligned} \tag{7}$$

(where Einstein's notation has been used) into Eq. (6), and we obtain:

$$\begin{aligned}
X_{t+\tau}^{h_i} - X^{h_i} = &\int_0^{\tau} \mu^{h_i}(\mathbf{X})d\tau' + \int_0^{\tau} \sigma_{h_i h_j}(\mathbf{X})dW_{\tau'}^{h_j} \\
&+ \int_0^{\tau} \left(\frac{\partial}{\partial X^{h_k}}\mu^{h_i}(\mathbf{X})\right)(X_{t+\tau'}^{h_k} - X^{h_k})d\tau' \\
&+ \int_0^{\tau} \left(\frac{\partial}{\partial X^{h_k}}\sigma_{h_i h_j}(\mathbf{X})\right)(X_{t+\tau'}^{h_k} - X^{h_k})dW_{\tau'}^{h_j} + \ldots
\end{aligned} \tag{8}$$

The first terms of this equation can be rewritten as:

$$X_{t+\tau}^{h_i} - X^{h_i} = \mu^{h_i}(\mathbf{X})\tau + \sigma_{h_i h_j}(\mathbf{X})W_{\tau}^{h_j} + \ldots \tag{9}$$

Iterating Eq. (8) by replacing $(X_{t+\tau'}^{h_k} - X^{h_k})$ with Eq. (9) we obtain:

$$\begin{aligned}
X_{t+\tau}^{h_i} - X^{h_i} = &\int_0^{\tau} \mu^{h_i}(\mathbf{X})d\tau' + \int_0^{\tau} \sigma_{h_i h_j}(\mathbf{X})dW_{\tau'}^{h_j} \\
&+ \int_0^{\tau} \left(\frac{\partial}{\partial X^{h_k}}\mu^{h_i}(\mathbf{X})\right)(\mu^{h_k}(\mathbf{X})\tau' + \sigma_{h_k h_j}(\mathbf{X})W_{\tau'}^{h_j} + \ldots)d\tau' \\
&+ \int_0^{\tau} \left(\frac{\partial}{\partial X^{h_k}}\sigma_{h_i h_j}(\mathbf{X})\right)(\mu^{h_k}(\mathbf{X})\tau' + \sigma_{h_k h_j}(\mathbf{X})W_{\tau'}^{h_j} + \ldots)dW_{\tau'}^{h_j} + \ldots
\end{aligned} \tag{10}$$

Finally, applying $\langle\cdot\rangle$ on both sides of Eq. (10), and keeping only terms proportional to τ:

$$\begin{aligned}
\langle X_{t+\tau}^{h_i} - X^{h_i} \rangle = &\mu^{h_i}(\mathbf{X})\tau + \sigma_{h_i h_j}(\mathbf{X})\langle W_{\tau}^{h_j} \rangle \\
&+ \left(\frac{\partial}{\partial X^{h_k}}\mu^{h_i}(\mathbf{X})\right)\sigma_{h_k h_j}(\mathbf{X})\left\langle \int_0^{\tau} W_{\tau'}^{h_j} d\tau' \right\rangle \\
&+ \left(\frac{\partial}{\partial X^{h_k}}\sigma_{h_i h_j}(\mathbf{X})\right)\sigma_{h_k h_j}(\mathbf{X})\left\langle \int_0^{\tau} W_{\tau'}^{h_j} dW_{\tau'}^{h_j} \right\rangle
\end{aligned} \tag{11}$$

Recall that $W_{\tau}^{h_j}$ is a Wiener process, therefore:

$$\begin{aligned}
\langle W_{\tau}^{h_j} \rangle &= 0 \\
\left\langle \int_0^{\tau} W_{\tau'}^{h_j} d\tau' \right\rangle &= 0 \\
\left\langle \int_0^{\tau} W_{\tau'}^{h_j} dW_{\tau'}^{h_j} \right\rangle &= 0
\end{aligned} \tag{12}$$

Thus, $\langle X^{h_i}_{t+\tau} - X^{h_i} \rangle = \mu^{h_i}(\mathbf{X})\tau$, which yields the following relationship between the first Kramers-Moyal coefficient and the drift:

$$D^{(1)}_{h_i}(\mathbf{X}, t) = \lim_{\tau \to 0} \frac{\langle X^{h_i}_{t+\tau} - X^{h_i} \rangle}{\tau} = \mu^{h_i}(\mathbf{X}) \tag{13}$$

Following the same rationale for $\langle (X^{h_i}_{t+\tau} - X^{h_i})(X^{h_j}_{t+\tau} - X^{h_j}) \rangle$, the second Kramers-Moyal coefficient keeps the following relationship with the diffusion term:

$$D^{(2)}_{h_i h_j}(\mathbf{X}, t) = \frac{1}{2} \lim_{\tau \to 0} \frac{\langle [X^{h_i}_{t+\tau} - X^{h_i}][X^{h_j}_{t+\tau} - X^{h_j}] \rangle}{\tau} = \frac{1}{2} \sigma_{h_i h_k}(\mathbf{X}) \sigma_{h_j h_k}(\mathbf{X}) \tag{14}$$

The remaining KM coefficients ($n \geq 3$) are zero, given that the stochastic process is Gaussian.

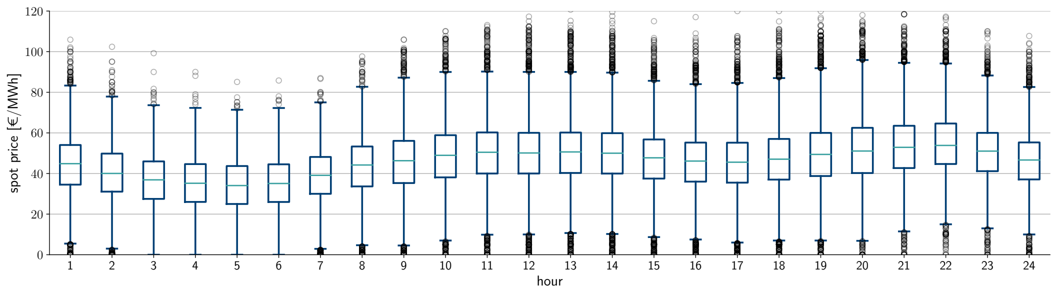

Figure 4: Hourly spot price boxplot for the Spanish day-ahead market. Period: 2004-2020.